Semi-Quantum Key Distribution with Limited Measurement Capabilities

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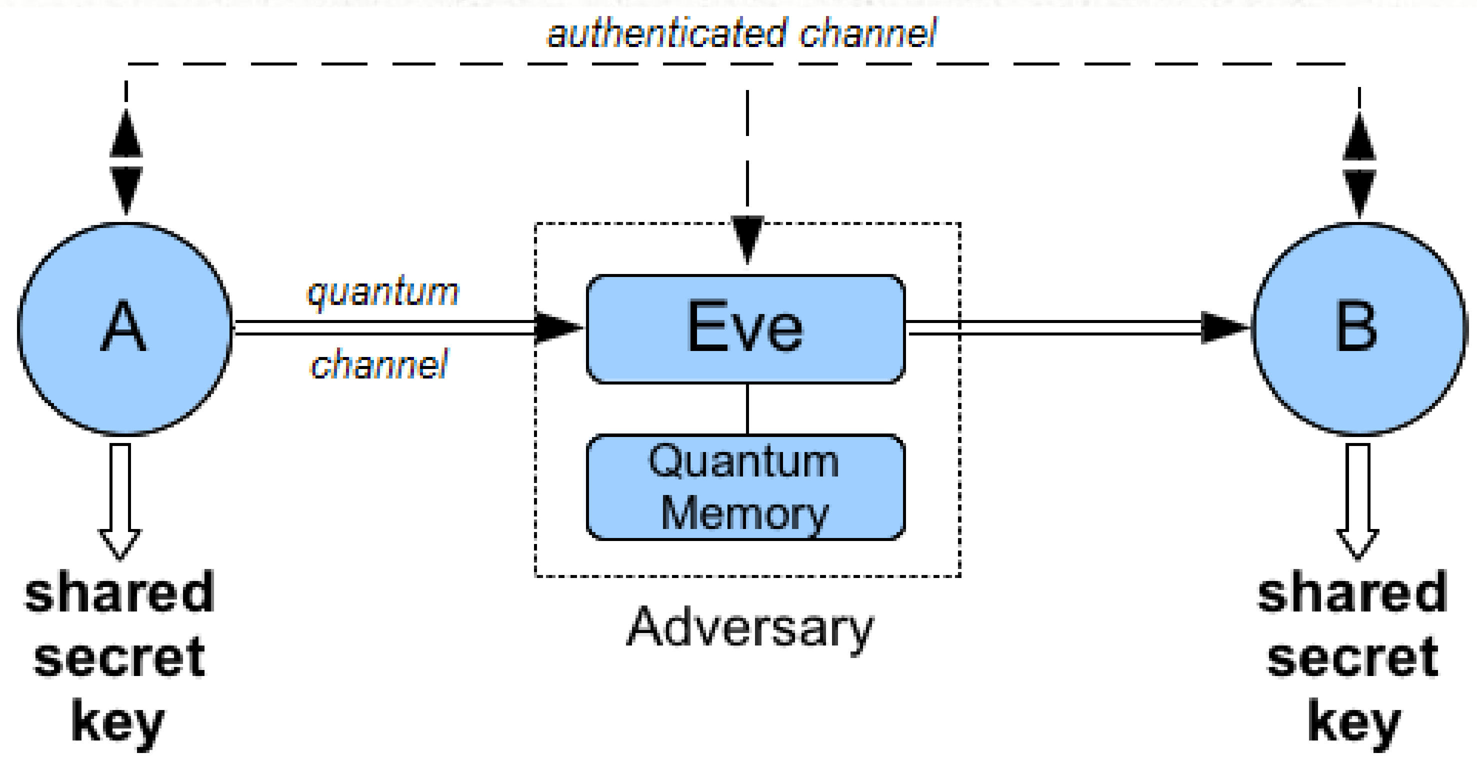
Quantum Key Distribution (QKD)

- Allows two users – Alice (A) and Bob (B) – to establish a shared secret key
- Secure against an all powerful adversary
  - Does not require any computational assumptions
  - Attacker bounded only by the laws of physics
  - Something that is not possible using classical means only
- Accomplished using a *quantum communication channel*

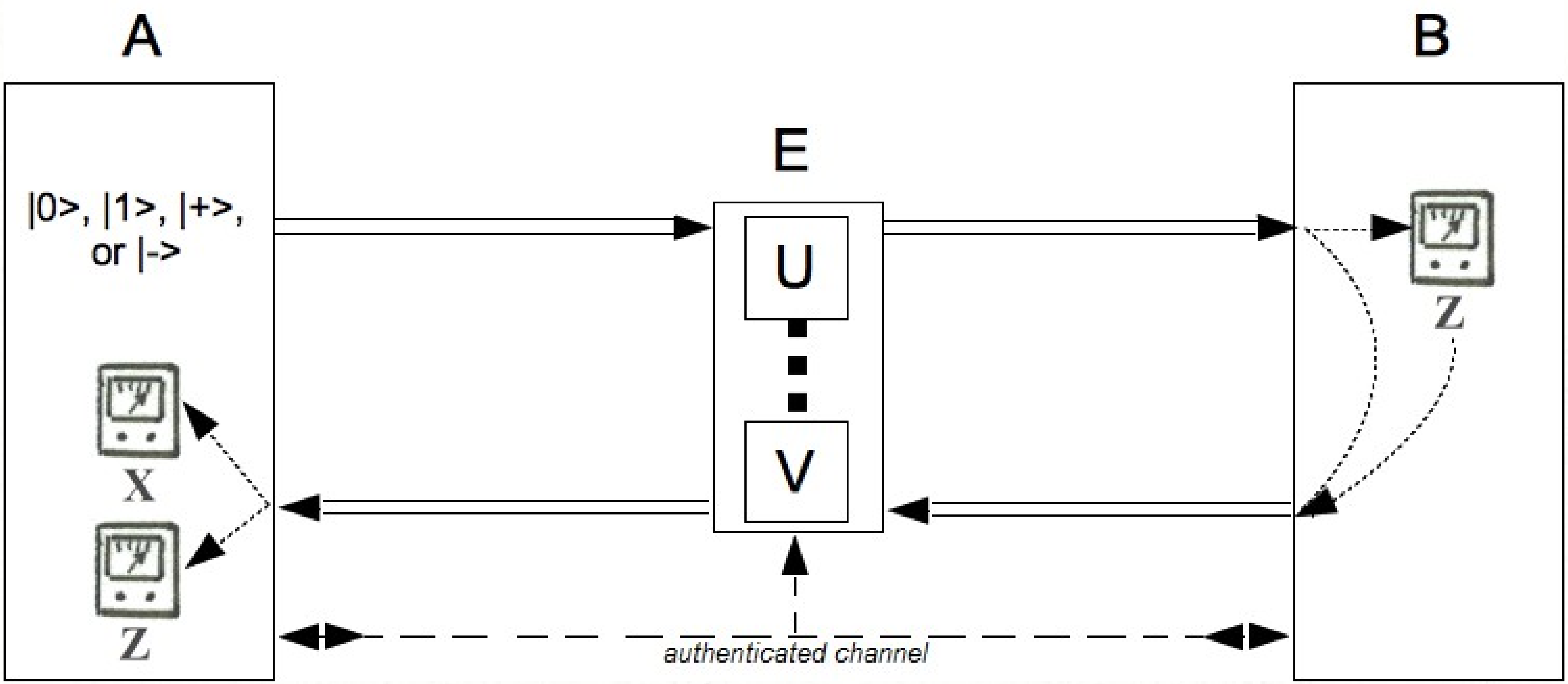
Quantum Key Distribution



Semi-Quantum Key Distribution

- In 2007, Boyer et al., introduced *semi-quantum key distribution* (SQKD)
- Now Alice (A) is quantum, but Bob (B) is limited or "classical"
    - He can only directly work with the Z = {|0>, |1>} basis.
- Theoretically interesting:
    - "How quantum does a protocol need to be in order to gain an advantage over a classical one?"
- Practically interesting:
    - What if equipment breaks down or is never installed?
- **Requires a two-way quantum communication channel**

Semi-Quantum Key Distribution



SQKD Security

- Model introduced in 2007, with many protocols developed
  - But security proofs were in terms of "robustness"
- Not until 2015 that rigorous security proofs became available for some protocols along with noise tolerances and key-rate bounds
  - Noise tolerance shown to be 6.1% if using only error-statistics
  - Tolerance is 11% if using **mismatched measurements** [5,9,10]
    - Requires 18 different measurement statistics

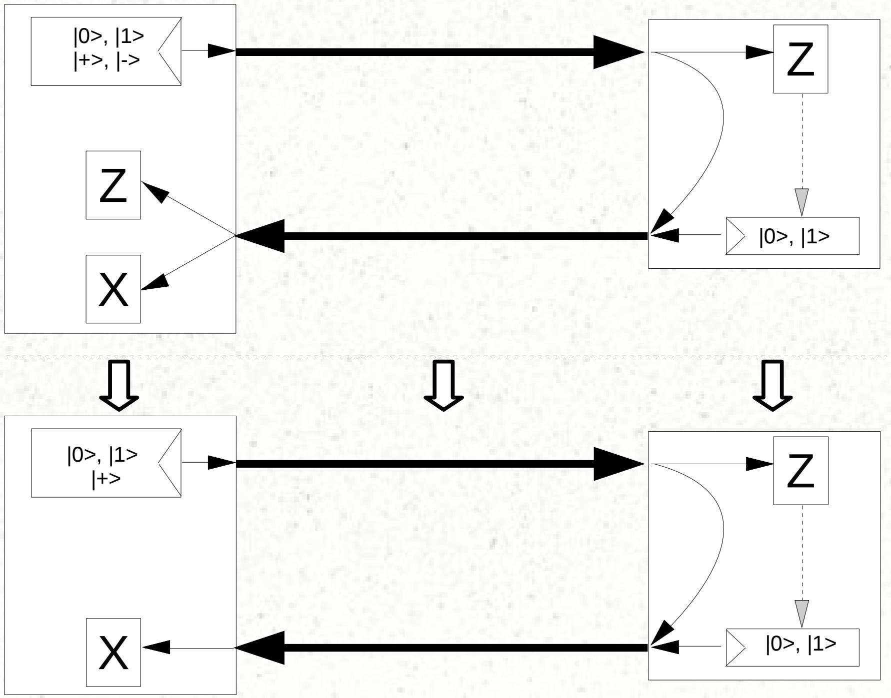
New Protocol

- All SQKD protocols require a two-way quantum channel
- All SQKD protocols so far have required the quantum user to measure in two (or more) bases
- We show this is not necessary
- Furthermore, the noise tolerance of our new protocol is just as high as BB84 assuming symmetric attacks!

New Protocol

Original:

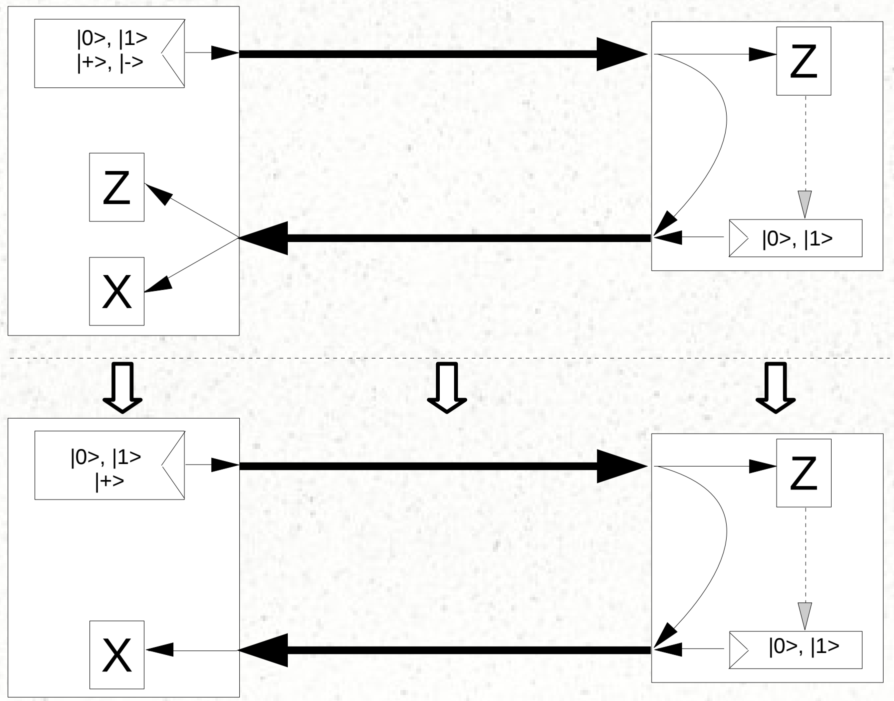
New:



New Protocol

Original:

New:



Interestingly, protocol is **insecure** if we only look at error rates – looking at mismatched measurements is **necessary** for security of this protocol!

Our Contributions

- We propose a new SQKD protocols where **both** users have severe restrictions placed on their measurement capabilities
- We show how the technique of **mismatched measurements** [9,10] can be applied to this two-way protocol to produce very optimistic key-rate bounds
  - We also show that it is necessary to look at these mismatched statistics!
- We show our protocol has the same noise tolerance as other SQKD and **fully-quantum** QKD protocols

[9] S. M. Barnett, B. Huttner, and S. J. Phoenix, “Eavesdropping strategies and rejected-data protocols in quantum cryptography,” Journal of Modern Optics, vol. 40, no. 12, pp. 2501–2513, 1993.

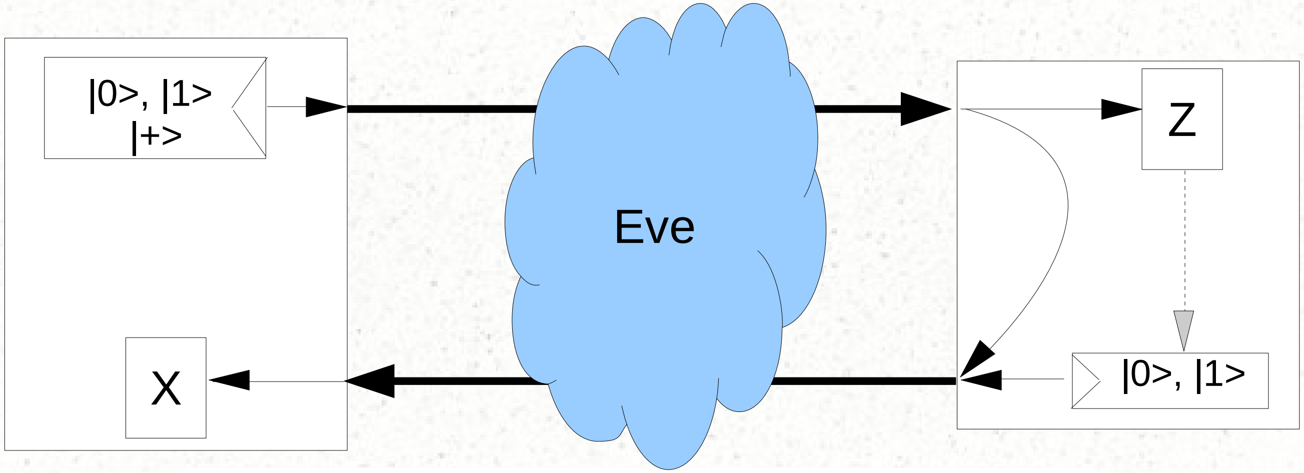
[10] S. Watanabe, R. Matsumoto, and T. Uyematsu, “Tomography increases key rates of quantum-key distribution protocols,” Physical Review A, vol. 78, no. 4, p. 042316, 2008.

The Protocol

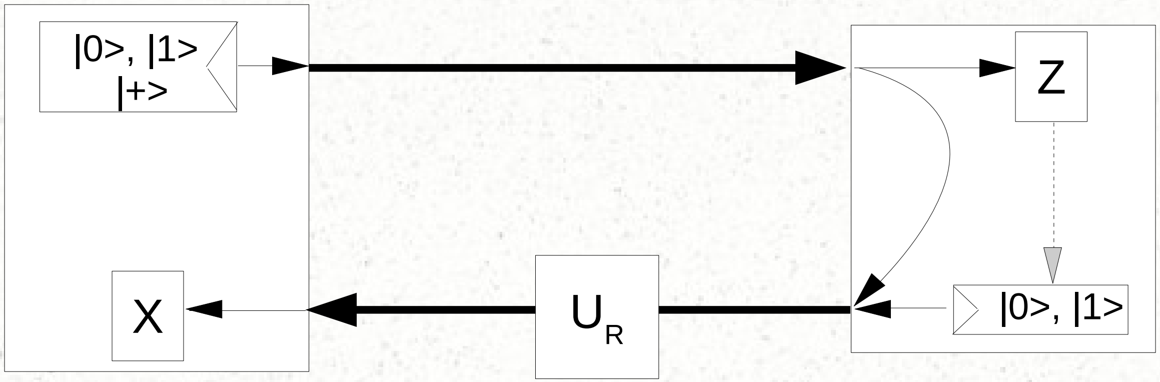
The Protocol

- Alice's Restrictions:
  - Can only send |0>, |1>, or |+>
  - Can only measure in the X basis {|+>, |->}
- Bob's Restrictions:
  - **Measure-and-Resend**: Measure in the Z basis and resend the observed result
  - **Reflect**: Disconnect from the quantum channel and ignore the incoming state

The Protocol (in a nutshell)



Need for Mismatched Measurements

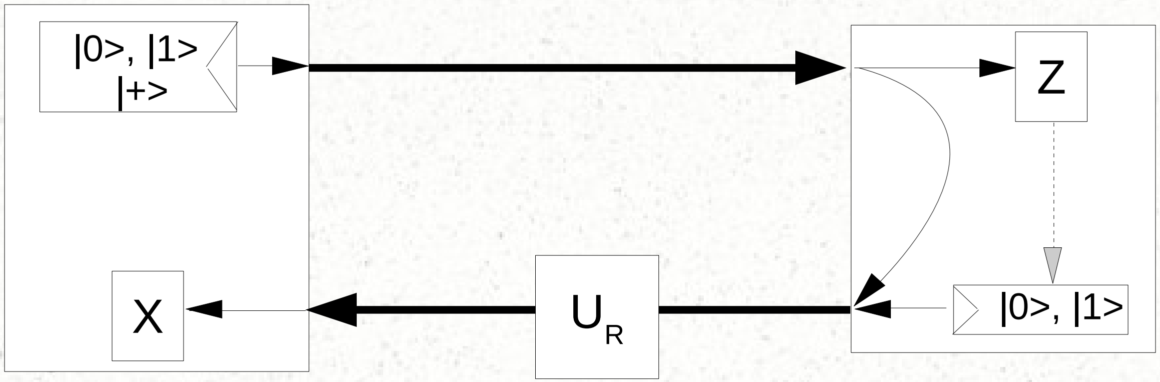


Forward Channel: Ignore (no noise)

Reverse Channel, apply U_R:

$$U_R|+>=|+,0>$$
$$U_R|->=|+,1>$$

Need for Mismatched Measurements



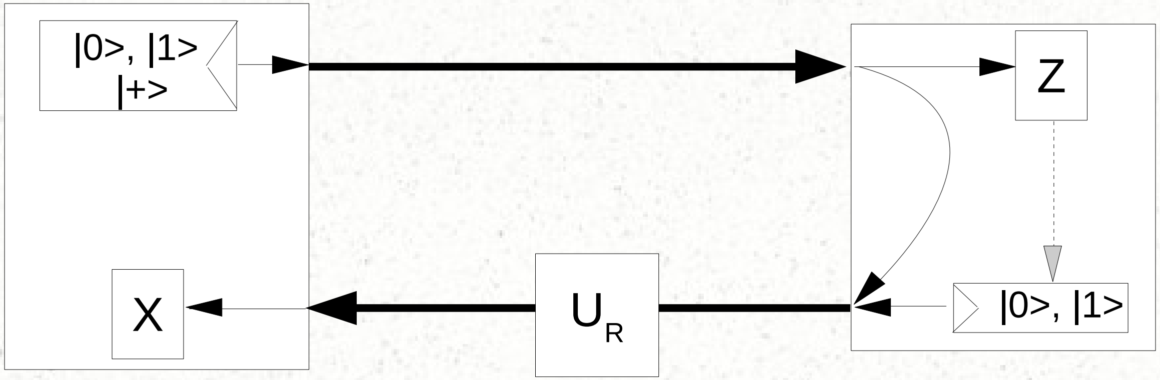
Forward Channel: Ignore (no noise)

Reverse Channel, apply U_R:

$$U_R|+>=|+,0>$$
$$U_R|->=|+,1>$$

No detectable noise!

Need for Mismatched Measurements

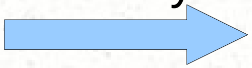


Forward Channel: Ignore (no noise)

Reverse Channel, apply U_R:

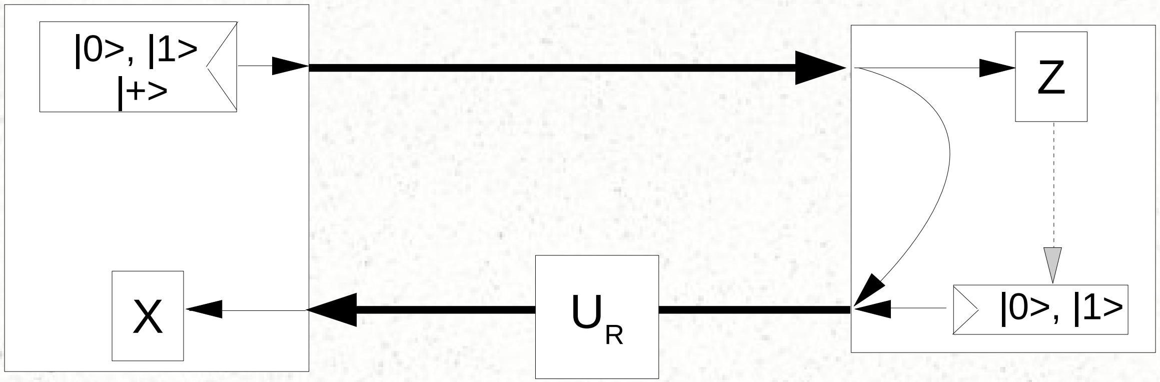
$$U_R|+\rangle = |+,0\rangle$$
$$U_R|-\rangle = |+,1\rangle$$

Linearity



$$U_R|0\rangle = |+,+\rangle$$
$$U_R|1\rangle = |+,-\rangle$$

Need for Mismatched Measurements



$$U_R|+\rangle=|+,0\rangle$$
$$U_R|-\rangle=|+,1\rangle$$

$$U_R|0\rangle=|+,+\rangle$$
$$U_R|1\rangle=|+,-\rangle$$

Two Fixes:

•Increase complexity of protocol by having A send |->

•Use **mismatched measurements** [5,9,10]

Security Proof

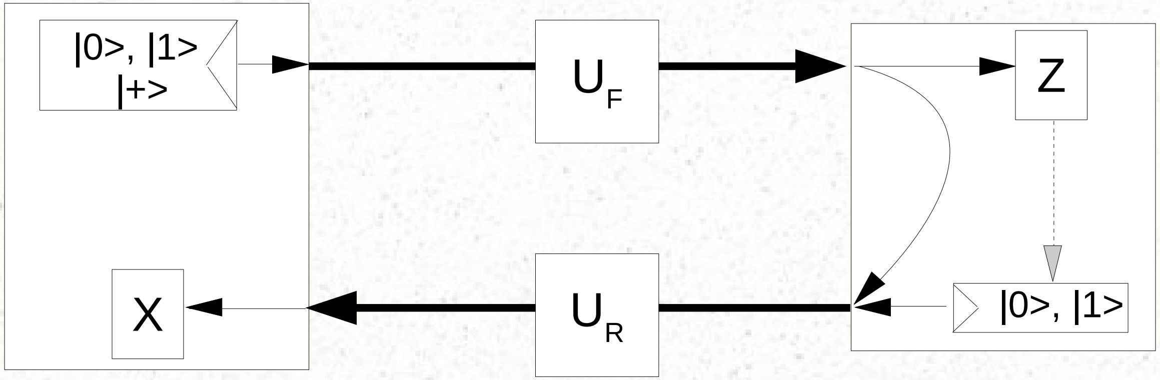
General QKD Security

- We consider collective attacks (and comment on general attacks later)
- After the quantum communication stage and parameter estimation stage, A and B hold an N bit raw key; E has a quantum system
- They then run an error correcting protocol and privacy amplification protocol
- Result is an l(n)-bit secret key – of interest is Devetak-Winter key-rate:

$$r = \lim_{N \to \infty} \frac{l(N)}{N} = inf\left(S(A|E) - H(A|B)\right)$$

Two Attacks

Eve is allowed to opportunities to probe the qubit:



Forward:
$$U_F|0,0>_{TE}=|0,e_0>+|1,e_1>$$
$$U_F|1,0>_{TE}=|1,e_2>+|1,e_3>$$

Reverse:
$$U_R|i,e_j>_{TE}=|0,e_{i,j}^0>+|1,e_{i,j}^1>$$

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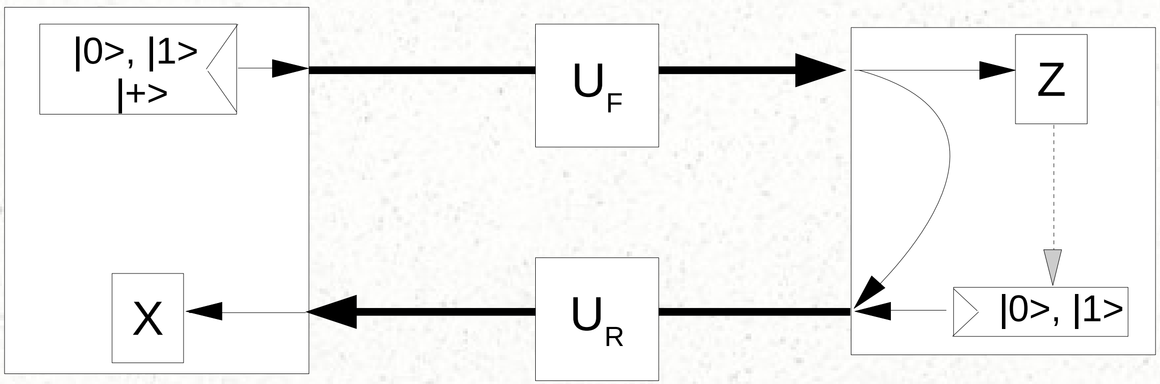

Forward:

$$U_F|0,0>_{TE}=|0,e_0>+|1,e_1>$$
$$U_F|1,0>_{TE}=|1,e_2>+|1,e_3>$$

Not necessarily normalized or orthogonal

Reverse:

$$U_R|i,e_j>_{TE}=|0,e^0_{i,j}>+|1,e^1_{i,j}>$$

Quantum State ABE

- With this notation, simple algebra allows us to derive the following density operator describing one iteration (conditioning on a key-bit being distilled):

$$\rho_{ABE} = \frac{1}{2}[0,0]_{AB} \otimes \left([e^{0}_{0,0}] + [e^{1}_{0,0}]\right) + \frac{1}{2}[0,1]_{AB} \otimes \left([e^{0}_{1,1}] + [e^{1}_{1,1}]\right)$$
$$+ \frac{1}{2}[1,0]_{AB} \otimes \left([e^{0}_{0,2}] + [e^{1}_{0,2}]\right) + \frac{1}{2}[1,1]_{AB} \otimes \left([e^{0}_{1,3}] + [e^{1}_{1,3}]\right)$$

Note: $[x] = |x\rangle\langle x|$

$$\rho_{ABE}=\frac{1}{2}[0,0]_{AB}\otimes([e^0_{0,0}]+[e^1_{0,0}])+\frac{1}{2}[0,1]_{AB}\otimes([e^0_{1,1}]+[e^1_{1,1}])$$
$$+\frac{1}{2}[1,0]_{AB}\otimes([e^0_{0,2}]+[e^1_{0,2}])+\frac{1}{2}[1,1]_{AB}\otimes([e^0_{1,3}]+[e^1_{1,3}])$$

Using a result in [5] allows us to bound:

$$S(A|E)\geq\frac{<e^0_{0,0}|e^0_{0,0}>+<e^1_{1,3}|e^1_{1,3}>}{2}\left(h\left(\frac{<e^0_{0,0}|e^0_{0,0}>}{<e^0_{0,0}|e^0_{0,0}>+<e^1_{1,3}|e^1_{1,3}>}\right)-h(\lambda_1)\right)$$
$$+\frac{<e^1_{0,0}|e^1_{0,0}>+<e^0_{1,3}|e^0_{1,3}>}{2}\left(h\left(\frac{<e^1_{0,0}|e^1_{0,0}>}{<e^1_{0,0}|e^1_{0,0}>+<e^0_{1,3}|e^0_{1,3}>}\right)-h(\lambda_2)\right)$$
$$+\frac{<e^1_{1,1}|e^1_{1,1}>+<e^0_{0,2}|e^0_{0,2}>}{2}\left(h\left(\frac{<e^1_{1,1}|e^1_{1,1}>}{<e^1_{1,1}|e^1_{1,1}>+<e^0_{0,2}|e^0_{0,2}>}\right)-h(\lambda_3)\right)$$
$$+\frac{<e^0_{1,1}|e^0_{1,1}>+<e^1_{0,2}|e^1_{0,2}>}{2}\left(h\left(\frac{<e^0_{1,1}|e^0_{1,1}>}{<e^0_{1,1}|e^0_{1,1}>+<e^1_{0,2}|e^1_{0,2}>}\right)-h(\lambda_4)\right)$$

Unlike past SQKD protocols, we can only bound these
(based on the noise in the **forward channel**)

$$S(A|E) \geq \frac{<e^0_{0,0}|e^0_{0,0}> + <e^1_{1,3}|e^1_{1,3}>}{2}\left(h\left(\frac{<e^0_{0,0}|e^0_{0,0}>}{<e^0_{0,0}|e^0_{0,0}> + <e^1_{1,3}|e^1_{1,3}>}\right) - h(\lambda_1)\right)$$

$$+ \frac{<e^1_{0,0}|e^1_{0,0}> + <e^0_{1,3}|e^0_{1,3}>}{2}\left(h\left(\frac{<e^1_{0,0}|e^1_{0,0}>}{<e^1_{0,0}|e^1_{0,0}> + <e^0_{1,3}|e^0_{1,3}>}\right) - h(\lambda_2)\right)$$

$$+ \frac{<e^1_{1,1}|e^1_{1,1}> + <e^0_{0,2}|e^0_{0,2}>}{2}\left(h\left(\frac{<e^1_{1,1}|e^1_{1,1}>}{<e^1_{1,1}|e^1_{1,1}> + <e^0_{0,2}|e^0_{0,2}>}\right) - h(\lambda_3)\right)$$

$$+ \frac{<e^0_{1,1}|e^0_{1,1}> + <e^1_{0,2}|e^1_{0,2}>}{2}\left(h\left(\frac{<e^0_{1,1}|e^0_{1,1}>}{<e^0_{1,1}|e^0_{1,1}> + <e^1_{0,2}|e^1_{0,2}>}\right) - h(\lambda_4)\right)$$

Function of $\Re < e^0_{0,0} | e^1_{1,3} >$

$$S(A|E) \geq \frac{< e^0_{0,0} | e^0_{0,0} > + < e^1_{1,3} | e^1_{1,3} >}{2} \left(h\left(\frac{< e^0_{0,0} | e^0_{0,0} >}{< e^0_{0,0} | e^0_{0,0} > + < e^1_{1,3} | e^1_{1,3} >} \right) - h(\lambda_1) \right)$$

$$+ \frac{< e^1_{0,0} | e^1_{0,0} > + < e^0_{1,3} | e^0_{1,3} >}{2} \left(h\left(\frac{< e^1_{0,0} | e^1_{0,0} >}{< e^1_{0,0} | e^1_{0,0} > + < e^0_{1,3} | e^0_{1,3} >} \right) - h(\lambda_2) \right)$$

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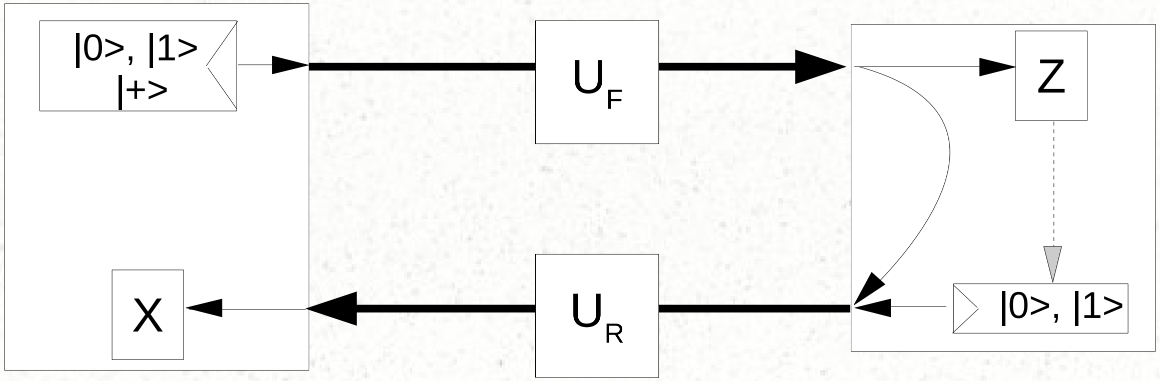
Parameter Estimation



Forward:

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Reverse:

$$U_R|i,e_j>_{TE}=|0,e^0_{i,j}>+|1,e^1_{i,j}>$$

$$p^{A\rightarrow B}_{0,0}=<e_0|e_0>$$

$$p^{A\rightarrow B}_{0,0}=<e^0_{0,0}|e^0_{0,0}>+<e^1_{0,0}|e^1_{0,0}>$$

Bound based on $p_{0,0}^{A\rightarrow B} = <e_{0,0}^{0}|e_{0,0}^{0}> + <e_{0,0}^{1}|e_{0,0}^{1}>$

$$S(A|E) \geq \frac{<e_{0,0}^{0}|e_{0,0}^{0}> + <e_{1,3}^{1}|e_{1,3}^{1}>}{2}\left(h\left(\frac{<e_{0,0}^{0}|e_{0,0}^{0}>}{<e_{0,0}^{0}|e_{0,0}^{0}> + <e_{1,3}^{1}|e_{1,3}^{1}>}\right) - h(\lambda_1)\right)$$

$$+\frac{<e_{0,0}^{1}|e_{0,0}^{1}> + <e_{1,3}^{0}|e_{1,3}^{0}>}{2}\left(h\left(\frac{<e_{0,0}^{1}|e_{0,0}^{1}>}{<e_{0,0}^{1}|e_{0,0}^{1}> + <e_{1,3}^{0}|e_{1,3}^{0}>}\right) - h(\lambda_2)\right)$$

$$+\frac{<e_{1,1}^{1}|e_{1,1}^{1}> + <e_{0,2}^{0}|e_{0,2}^{0}>}{2}\left(h\left(\frac{<e_{1,1}^{1}|e_{1,1}^{1}>}{<e_{1,1}^{1}|e_{1,1}^{1}> + <e_{0,2}^{0}|e_{0,2}^{0}>}\right) - h(\lambda_3)\right)$$

$$+\frac{<e_{1,1}^{0}|e_{1,1}^{0}> + <e_{0,2}^{1}|e_{0,2}^{1}>}{2}\left(h\left(\frac{<e_{1,1}^{0}|e_{1,1}^{0}>}{<e_{1,1}^{0}|e_{1,1}^{0}> + <e_{0,2}^{1}|e_{0,2}^{1}>}\right) - h(\lambda_4)\right)$$

Similarly, we can look at: $p_{i,j}^{A\to B}$

$$S(A|E)\geq\frac{<e_{0,0}^0|e_{0,0}^0>+<e_{1,3}^1|e_{1,3}^1>}{2}\left(h\left(\frac{<e_{0,0}^0|e_{0,0}^0>}{<e_{0,0}^0|e_{0,0}^0>+<e_{1,3}^1|e_{1,3}^1>}\right)-h(\lambda_1)\right)$$

$$+\frac{<e_{0,0}^1|e_{0,0}^1>+<e_{1,3}^0|e_{1,3}^0>}{2}\left(h\left(\frac{<e_{0,0}^1|e_{0,0}^1>}{<e_{0,0}^1|e_{0,0}^1>+<e_{1,3}^0|e_{1,3}^0>}\right)-h(\lambda_2)\right)$$

$$+\frac{<e_{1,1}^1|e_{1,1}^1>+<e_{0,2}^0|e_{0,2}^0>}{2}\left(h\left(\frac{<e_{1,1}^1|e_{1,1}^1>}{<e_{1,1}^1|e_{1,1}^1>+<e_{0,2}^0|e_{0,2}^0>}\right)-h(\lambda_3)\right)$$

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Just leaves: $\Re < e_{0,0}^{0} | e_{1,3}^{1} >$

$$S(A|E) \geq \frac{<e_{0,0}^{0}|e_{0,0}^{0}> + <e_{1,3}^{1}|e_{1,3}^{1}>}{2} \left(h\left(\frac{<e_{0,0}^{0}|e_{0,0}^{0}>}{<e_{0,0}^{0}|e_{0,0}^{0}> + <e_{1,3}^{1}|e_{1,3}^{1}>} \right) - h(\lambda_1) \right)$$

$$+ \frac{<e_{0,0}^{1}|e_{0,0}^{1}> + <e_{1,3}^{0}|e_{1,3}^{0}>}{2} \left(h\left(\frac{<e_{0,0}^{1}|e_{0,0}^{1}>}{<e_{0,0}^{1}|e_{0,0}^{1}> + <e_{1,3}^{0}|e_{1,3}^{0}>} \right) - h(\lambda_2) \right)$$

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Parameter Estimation

However, we show that techniques applying mismatched measurements for two-way semi-quantum protocols derived in [5] can be applied to this scenario.

By looking at the error-rate in the "reflection" case, we find:

$$p_{+,R,-}^{A \to A} = 1 - \frac{1}{2}(L_1 + L_2 + L_3 + L_4 + \eta_1 + \eta_2) - \frac{1}{2}(p_{0,R,+}^{A \to A} + p_{1,R,+}^{A \to A})$$

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Needed to compute λ_i

e.g., $L_1 = \Re < e_{0,0}^0 | e_{1,3}^1 >$

Parameter Estimation

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Mismatched Measurements – in a symmetric attack, these are ½ each

Parameter Estimation

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Functions of five different mismatched statistics (each).

If symmetric attack, it holds that: $\eta_1 = \eta_2 = 0$

Entropy Computation

- Our entropy bound on S(A|E) is a function of eight variables:

$$<e^1_{0,0}|e^1_{0,0}>, <e^1_{1,3}|e^1_{1,3}>, <e^1_{0,2}|e^1_{0,2}>, <e^1_{1,1}|e^1_{1,1}>, L_1, L_2, L_3, L_4$$

- With restrictions:

Restriction	Reason
$<e^k_{i,j}\|e^k_{i,j}> \geq 0$	Property of inner-product
$<e^1_{0,0}\|e^1_{0,0}> \leq p^{A\to B}_{0,0}$ $<e^1_{1,3}\|e^1_{1,3}> \leq p^{A\to B}_{1,1}$ $<e^1_{0,2}\|e^1_{0,2}> \leq p^{A\to B}_{1,0}$ $<e^1_{1,1}\|e^1_{1,1}> \leq p^{A\to B}_{0,1}$	Unitarity of U_R
$\|L_1\| \leq \sqrt{<e^0_{0,0}\|e^0_{0,0}><e^1_{1,3}\|e^1_{1,3}>}$ $\|L_2\| \leq \sqrt{<e^1_{0,0}\|e^1_{0,0}><e^0_{1,3}\|e^0_{1,3}>}$ $\|L_3\| \leq \sqrt{<e^1_{1,1}\|e^1_{1,1}><e^0_{0,2}\|e^0_{0,2}>}$ $\|L_4\| \leq \sqrt{<e^0_{1,1}\|e^0_{1,1}><e^1_{0,2}\|e^1_{0,2}>}$	Cauchy-Schwarz
$p^{A\to A}_{+,R,-} = 1 - \frac{1}{2}(L_1 + L_2 + L_3 + L_4 + \eta_1 + \eta_2)$ $- \frac{1}{2}(p^{A\to A}_{0,R,+} + p^{A\to A}_{1,R,+})$	Mismatched Measurements

Evaluation + Summary

Results

- We numerically minimize S(A|E) based on the above constraints
  - Need to minimize as we must assume the worst case
- Computing H(A|B) is trivial given observable data
- Thus, we can compute the key-rate r = S(A|E) - H(A|B)

	Independent: $Q_X = 2Q(1-Q)$	Dependent: $Q_X = Q$
Max. Q:	$Q < 7.9\%$	$Q < 11\%$

Required Measurement Statistics

Error Rates

$p_{0,0}^{A \to B}$
$p_{0,1}^{A \to B}$
$p_{1,0}^{A \to B}$
$p_{1,1}^{A \to B}$

$p_{+,R,-}^{A \to A}$

Mismatched Events

$p_{+,0}^{A \to B}$
$p_{+,1}^{A \to B}$

$p_{0,R,+}^{A \to A}$
$p_{1,R,+}^{A \to A}$
$p_{+,0,+}^{A \to A}$
$p_{0,0,+}^{A \to A}$
$p_{1,0,+}^{A \to A}$
$p_{+,1,+}^{A \to A}$
$p_{0,1,+}^{A \to A}$
$p_{1,1,+}^{A \to A}$

Required Measurement Statistics

Error Rates

$p_{0,0}^{A \to B}$

$p_{0,1}^{A \to B}$

$p_{1,0}^{A \to B}$

$p_{1,1}^{A \to B}$

$p_{+,R,-}^{A \to A}$

Mismatched Events

$p_{+,0}^{A \to B}$

$p_{+,1}^{A \to B}$

$p_{0,R,+}^{A \to A}$

$p_{1,R,+}^{A \to A}$

$p_{+,0,+}^{A \to A}$

$p_{0,0,+}^{A \to A}$

$p_{1,0,+}^{A \to A}$

$p_{+,1,+}^{A \to A}$

$p_{0,1,+}^{A \to A}$

$p_{1,1,+}^{A \to A}$

While we only evaluated on a symmetric channel, our equations apply to arbitrary channels.

Future Work

- How does the protocol compare to others over non-symmetric attacks?
- We only considered collective attacks – does the usual techniques of applying de Finetti work here?
    - Or some other way to extend to general attacks
- What about a finite-key analysis?
    - Especially comparing with other SQKD or fully quantum protocols.

Thank you! Questions?

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